Int. J. Heat Mass Transfer. Vol. 23, pp. 45–51.
Pergamon Press Ltd. 1980. Printed in Great Britain

LAMINAR NATURAL CONVECTION HEAT TRANSFER FROM AN INCLINED CYLINDER

M. AL-ARABI* and Y. K. SALMAN
Mechanical Engineering Department, University of Technology, Baghdad, Iraq

(*Received* 27 *July* 1978 *and in revised form* 26 *January* 1979)

Abstract—Laminar natural convection heat transfer from the outside surface of a uniformly heated cylinder (constant heat flux condition) was investigated experimentally at different angles of inclination of the cylinder. General equations for the effect of inclination were determined for both the local and the average heat transfer.

NOMENCLATURE

D, outside diameter of cylinder [m];
h, coefficient of heat transfer [W/m^2 °C];
L, length of cylinder [m];
q, heat flux [W/m^2];
t, temperature [°C];
x, distance from beginning of cylinder [m];
Gr, Grashoff number;
Nu, Nusselt number;
Pr, Prandtl number;
θ, angle of inclination of cylinder.

Subscripts

a, ambient;
cr, critical;
x, local (at length of cylinder = x);
s, surface;
L, average (for a length of cylinder = L).

INTRODUCTION

MANY investigations were carried out on natural convection heat transfer from the outside surface of vertical and horizontal cylinders in both the constant temperature and the constant heat flux conditions. The data on natural convection from inclined cylinders are, however, very limited.

The present research work was carried out to fill a part of the existing gap. It is concerned mainly with the study of the effect of the angle of inclination on natural convection heat transfer from the outside surface of a cylinder in the constant heat flux condition.

PREVIOUS WORK

As far the authors know only three investigations are available so far in the field of inclined cylinders. They are all experimental but the conditions in which they were carried out are not the same. As will be seen only one of them was carried out in the constant heat flux condition [1]. Of the other two, one was carried out in the constant temperature condition [2] and the other was carried out "in what was considered to be a constant temperature condition" [3].

Farber and Rennat [1], in 1957, experimented with a stainless steel tube 6 ft (1·829 m) long and 0·125 in (3·175 mm) OD heated by passing an electric current through it to give a constant heat flux. Temperatures as high as 760°C were obtained and the angle of inclination of the tube to the horizontal was varied from 0 to 90°.

Khamis [2], in 1975, experimented with steam heated (constant temperature) brass tubes of different lengths and diameters. The range of $Gr_L Pr$ was varied from $9{\cdot}88 \times 10^7$ to $2{\cdot}93 \times 10^{10}$ and the angle of inclination of the tube to the horizontal from 30° to 90°.

Oosthuizen [3], in 1976, experimented with aluminium cylinders. The diameters varied between 19·1 and 25·4 mm, the length between 152·4 and 304·8 mm and the angle of inclination to the horizontal between 0 and 90°. The heat transfer was determined by measuring the rate at which the cylinders were cooled (to 90°C) after being uniformly heated (to 100°C).

APPARATUS

The experimental apparatus is shown diagramatically in Fig. 1. It consists essentially of a copper cylinder A of a length equal to 950 mm, OD equal to 38 mm and ID equal to 36 mm.

Two teflon circular pieces B and C, also of OD equal to 38 mm, are fitted to the lower and upper ends of the cylinder so that the outside surface of the teflon forms a continuation of the copper surface. Each teflon piece is 120 mm long, a length sufficient to enable free flow of convection currents. Teflon was chosen because of its low thermal conductivity in order to reduce the heat loss from the cylinder ends.

The cylinder components (teflon lower piece + copper cylinder + teflon upper piece) are held by two plates K_1 and K_2 tied together by tie bolts D and are mounted on a wooden board E which can be rotated around a horizontal spindle. The inclination of the cylinder can thus be adjusted as required. To minimize the effect of the plates K_1 and K_2 on the

*Present address: Professor and Head of Mechanical Engineering Department, Al-Azhar University, Cairo, Egypt.

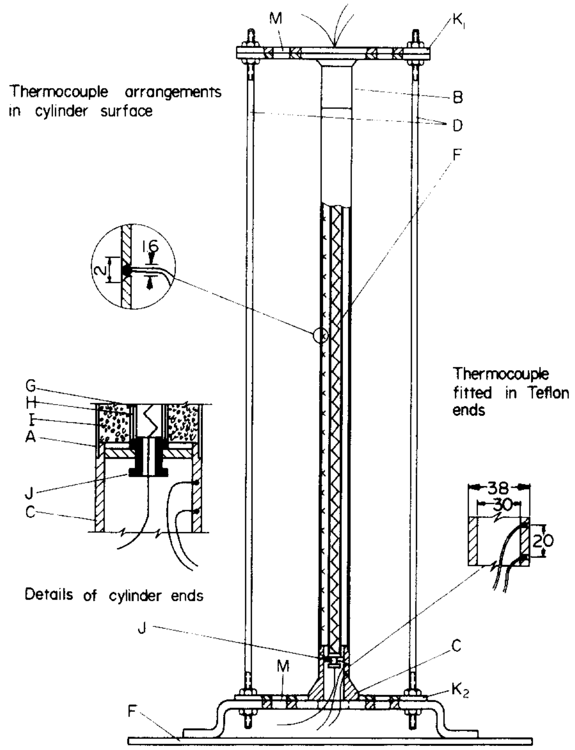

FIG. 1. Apparatus.

flow of convection currents holes M are drilled in them. In fact the effect of these plates was practically negligible as proved by the very good agreement between the experimental results for the vertical and horizontal cylinders and the more reliable published data.

The inside surface of the cylinder is heated by means of a 'bath-room' type electric heater. It consists of a nickel–chrome wire coil F uniformly wound to give constant heat generation per unit length. The coil is protected by an envelope of pyrex-glass tubing G. To avoid convection currents in the space between the glass tube and the cylinder this space is filled with asbestos partly as tape H wound on the tube and partly as asbestos powder I. The ends of the heater wire are connected to circular porecelain connection pieces J. The porcelain pieces are fitted in holes bored centrally in the teflon ends and therefore keep the heater concentric with the cylinder.

The temperature of the outside surface of the cylinder is measured by thirty-four 0·3 mm glass-covered copper–constantan thermocouples fitted along the cylinder. The measuring junctions were made by fusing the ends of the two wires together by an electric spark in an atmosphere free from oxygen.

To fix the thermocouples holes of 1·6 mm dia were drilled in the cylinder wall and the ends of the holes chamfered by a 2 mm drill to locate the measuring junctions which were then soft-soldered to the surface. The excess solder was removed and the surface cleaned carefully by fine grinding paper.

At two cylinder lengths (30 and 60 cm) four thermocouples are fitted around the cylinder circumference to make sure that the temperature at any point on the circumference is the same. No difference between the readings of the thermocouples at the same length was noted during the experiments.

The ambient temperature of air was measured by a thermocouple placed far from the effect of the hot cylinder.

To determine the heat loss by conduction from the cylinder ends to the teflon pieces two thermocouples were inserted in each teflon piece as shown in Fig. 1. From the thermal conductivity of teflon and the distance between the thermocouples the end conduction could be calculated. The maximum value of this in the experiments was less than 0·03% of the total heat given.

To minimize radiation from the cylinder it is nickel-electroplated and polished. The emissivity of polished nickel is taken as 0·072 [4]. The heat loss by radiation thus calculated varied between 4 and 10% of the total heat transfer. As the emissivity of "nickel not polished", as quoted by the same reference, is 0·11, the error which may result if the assumed value is on the low side can only be small.

To avoid the effects of air currents the cylinder is protected by shields placed at sufficient distance from the cylinder so as not to interfere with the free flow of convection currents.

The power consumed by the heater was measured by accurate ammeters and voltmeters, and a voltage stabilizer was used to ensure that the incoming mains voltage is constant. A potentiometer capable of reading 0·001 mV was used to take the readings of the thermocouples.

DEFINITIONS

The local heat-transfer coefficient h_x at a cylinder length equal to x is calculated from:

$$h_x = \frac{q}{(t_s) - t_a} \tag{1}$$

The average heat-transfer coefficient for a cylinder of length equal to L may be calculated from,

$$(h_L)_1 = \frac{q}{t_L - t_a} \tag{2}$$

where

$$t_L = \frac{I}{L}\int_{x=0}^{x=L} (t_s)_x \, dx \tag{3}$$

or from

$$(h_L)_2 = \frac{I}{L}\int_{x=0}^{x=L} h_x \, dx = \frac{I}{L}\int_{x=0}^{x=L} \frac{q}{(t_s)_x - t_a}. \tag{4}$$

The heat-transfer coefficients used for the correlations given in this paper are based on equation (2). However equation (2) has been found to give practically the same values as those given by equation (4). The equations obtained for the average heat transfer are, therefore, valid whether $(h_L)_1$ or $(h_L)_2$ is used.

The physical properties are taken at the mean film temperature $t_f = (I/2)(t_s + t_a)$ except the coefficient of expansion which is taken at t_a.

RESULTS AND DISCUSSION

Range of experiments

Some 50 runs were carried out with the cylinder in the vertical position and at angles of inclination of 30, 45, 60 and 90° (horizontal) from the vertical. Fluxes up to 1000 W/m^2 were used giving local surface temperatures up to 146°C and local values of $Gr\,Pr$ up to $6{\cdot}5 \times 10^9$.

Effect of angle of inclination. Figure 2 shows that at the same x, h_x is minimum for the vertical cylinder and that it increases with the angle of inclination. For the horizontal cylinder its value is maximum.

Critical length. The length of the laminar region (normally called the critical length), expressed as Gr_xPr, is practically constant for the same angle of inclination but it increases with this angle as shown in Table 1. In the horizontal position the critical length must be infinity as the whole cylinder is then in the laminar region.

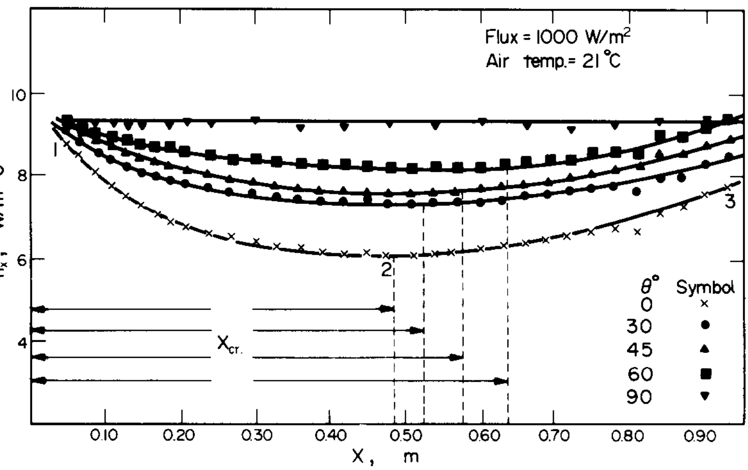

FIG. 2. Variation of local heat-transfer coefficient with cylinder length.

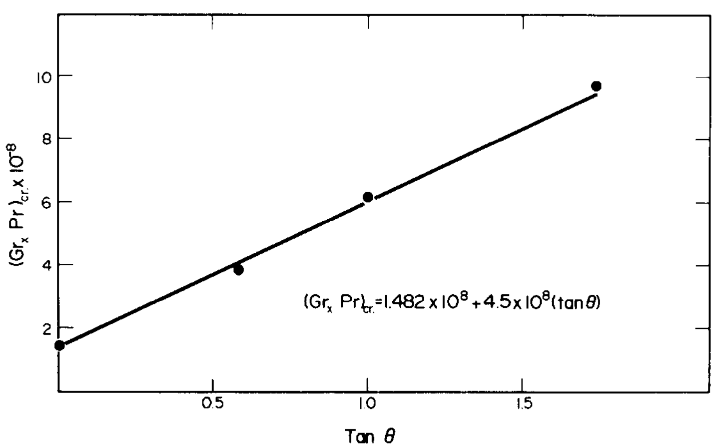

FIG. 3. Variation of $(Gr_xPr)_{cr}$ with $(\tan\theta)$.

Local heat transfer

Variation of local heat transfer with cylinder length. The variation of the local heat-transfer coefficient h_x with cylinder length for the different angles of inclination employed is plotted for the runs made with a flux equal to 1000 W/m^2 in Fig. 2. All the other fluxes give the same characteristics. As can be seen h_x decreases from point 1 until point 2 and then increases indicating a laminar region between points 1 and 2 and a transition region beyond that. As x approaches zero h_x must approach infinity since at $x = 0$ the boundary-layer thickness is zero. For the horizontal position h_x, as it should be, is constant.

Table 1.

Angle of inclination	$(Gr_xPr)_{cr}$
0° (vertical)	$1{\cdot}48 \times 10^8$
30°	$3{\cdot}85 \times 10^8$
45°	$6{\cdot}11 \times 10^8$
60°	$9{\cdot}70 \times 10^8$
90° (horizontal)	infinity

In an attempt to find a relation between the critical length and the angle of inclination the values of $(Gr_xPr)_{cr}$ are plotted against $\tan\theta$ in Fig. 3. As

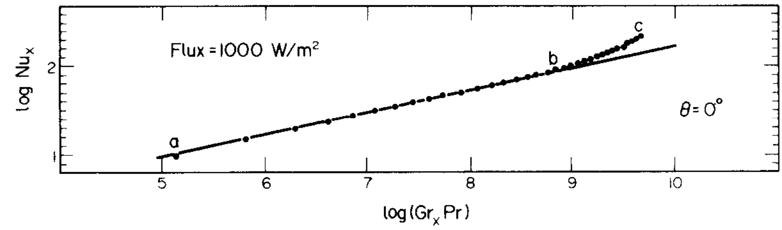

FIG. 4. Variation of Nu_x with $Gr_x Pr$.

can be seen a straight line relation is obtained the equation of which is:

$$(Gr_x Pr)_{cr} = 1{\cdot}48 \times 10^8 + 4{\cdot}5 \times 10^8 (\tan\theta). \tag{5}$$

Correlation of local heat-transfer data. The variation of the local Nusselt number Nu_x with the local $Gr_x Pr$ is shown in Fig. 4 in the form of log Nu_x against log $Gr_x Pr$ for the vertical position and a flux equal to 1000 W/m^2. The other angles and the other fluxes give the same characteristics. Between points a and b the results are represented by a straight line. Beyond point b the results follow a curve. The region a–b corresponds to the region 1–2 in Fig. 2 (laminar region) whereas the region b–c corresponds to the region 2–3 (transition region). The straight line a–b is represented by an equation of the following form for all the angles of inclination:

$$Nu_x = C(Gr_x Pr)^n. \tag{6}$$

The values of C and n for the different angles of inclination are shown in Table 2. For the horizontal position n must be equal to 1/3 since h_x is independent of x.

Table 2

Angle of inclination	C	n
0°	0·545	1/4
30°	0·405	0·275
45°	0·320	0·295
60°	0·219	0·315
90°	0·158	1/3

General equation for local heat transfer. Taking into consideration that n is equal to 1/4 for the vertical position and equal to 1/3 for the horizontal position it was assumed that it can be represented by the equation:

$$n = 1/4 + 1/12(\sin\theta)^s. \tag{7}$$

The fact that a plot between log $(n - 1/4)$ and log $(\sin\theta)$ gives a straight line (Fig. 5) shows that the form assumed is correct. The slope of this line is 1·75 and therefore n for all angles of inclination is represented by:

$$n = 1/4 + 1/12(\sin\theta)^{1{\cdot}75}. \tag{8}$$

A similar procedure gives Fig. 6 showing that C can be represented by:

$$C = 0{\cdot}545 - 0{\cdot}387(\sin\theta)^{1{\cdot}462}. \tag{9}$$

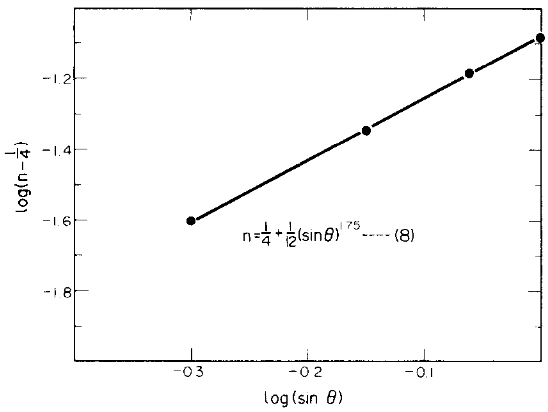

FIG. 5. Variation of log$(n - 1/4)$ with log(sin θ).

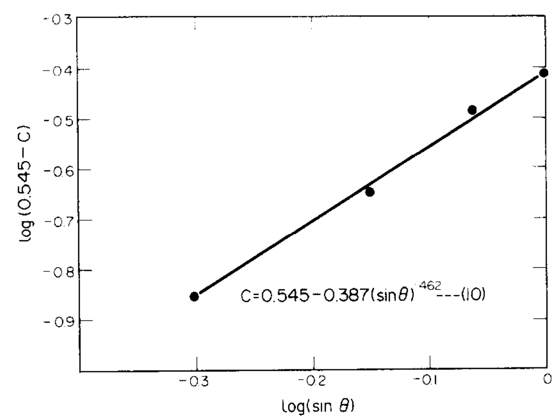

FIG. 6. Variation of log$(0{\cdot}545 - C)$ with log(sin θ).

Inserting the value of n from equation (8) and the value of C from equation (9) in equation (6) equation (10) is obtained. It represents the local laminar heat transfer for all the angles of inclination from 0° (vertical) to 90° (horizontal) for values of $Gr_x Pr$ less than the critical value as given by equation (5).

$$Nu_x = 0{\cdot}545 - 0{\cdot}387(\sin\theta)^{1{\cdot}462}(Gr_x Pr)^{1/4 + 1/12(\sin\theta)^{1{\cdot}75}} \tag{10}$$

The experimental data are plotted in Fig. 7 for comparison with equation (10). As can be seen they are all represented by this equation within ±4%.

Average heat transfer

Variation of average heat-transfer coefficient with cylinder length. The variation of the average heat-transfer coefficient h_L with cylinder length for the same runs shown in Fig. 2 is shown in Fig. 8. The

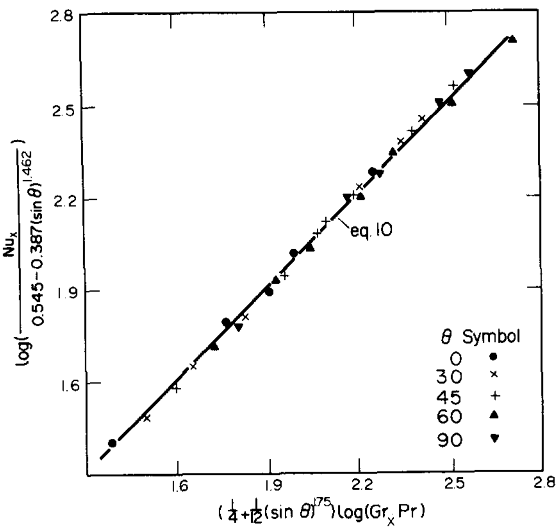

FIG. 7. Correlation of local heat-transfer data according to equation (10).

average heat-transfer coefficient decreases with cylinder length asymptotically for the angles 0°, 30°, 45°, and 60°. As L approaches zero h_L must approach infinity since at $L = 0$ the boundary-layer thickness is zero. For the horizontal position, however, h_L is constant.

Effect of angle of inclination. Figure 8 shows that at the same value of L, h_L is minimum for the vertical cylinder and that it increases with the angle of inclination. For the horizontal cylinder its value is maximum.

Correlation of average heat-transfer data. The variation of the average Nusselt number Nu_L with the average $Gr_L Pr$ is shown in Fig. 9 for the vertical position in the form of log Nu_L against log $(Gr_L Pr)$. The other angles of inclination give the same characteristics. As can be seen a straight line is obtained showing that:

$$Nu_L = K(Gr_L Pr)^m. \tag{11}$$

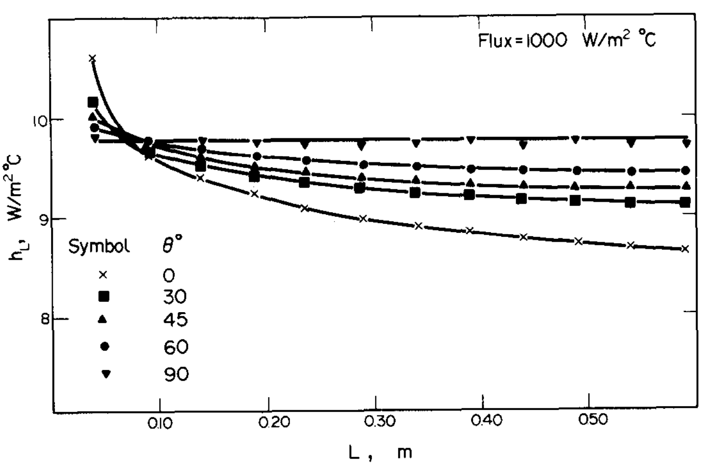

FIG. 8. Variation of average heat-transfer coefficient with cylinder length.

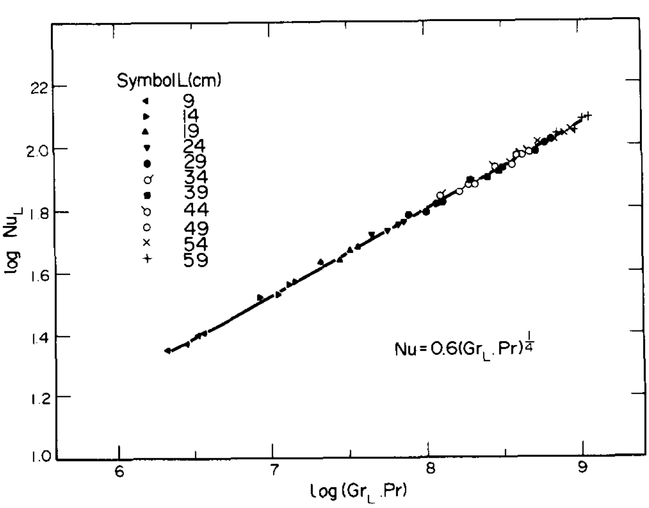

FIG. 9. Variation of Nu_L with $Gr_L Pr$.

The values of K and m for the different angles of inclination are given in Table 3.

Table 3

Angle of inclination	K	m
0°	0·60	1/4
30°	0·422	0·275
45°	0·335	0·295
60°	0·229	0·315
90°	0·158	1/3

Since for the horizontal position $h_L = h_x =$ constant the values of K and m [in equation (11)] must obviously be equal to the values of C and n respectively [in equation (6)].

General equation for average heat transfer. From Tables 2 and 3 it can be seen that the variation of m and n with the angle of inclination is the same for both the local heat transfer and the average heat transfer. As a result the relation between m and θ is the same as the relation between n and θ as given by equation (8).

A procedure similar to that used to determine the relation between C and θ in the local heat transfer shows that the relation between K and θ can be represented by:

$$K = 0{\cdot}60 - 0{\cdot}488(\sin\theta)^{1{\cdot}03}. \qquad (12)$$

The general equation for the average heat transfer for all the angles of inclination from 0° (vertical) to 90° (horizontal) and values of $Gr_x Pr$ less than the critical is therefore:

$$Nu_L = 0{\cdot}60 - 0{\cdot}488(\sin\theta)^{1{\cdot}03}(Gr_L Pr)^{1/4 + 1/12(\sin\theta)^{1{\cdot}75}} \qquad (13)$$

The experimental data for the different angles of inclination are plotted in Fig. 10 for comparison with equation (13). As can be seen they are all represented by this equation within $\pm 4{\cdot}5\%$.

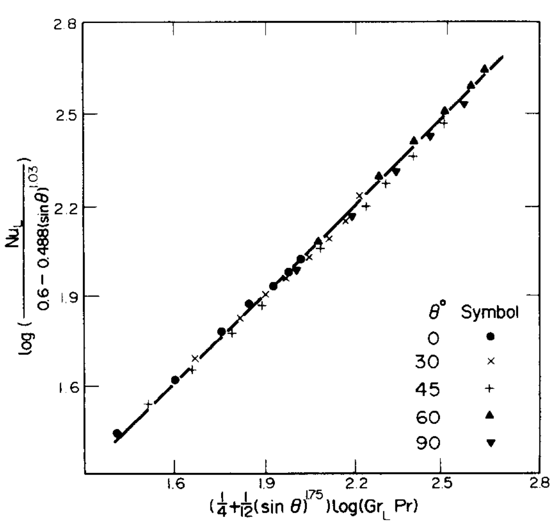

FIG. 10. Correlation of average heat-transfer data according to equation (13).

COMPARISON WITH PREVIOUS WORK

It is of interest to compare the equations obtained for the vertical and horizontal cylinders with those generally accepted at the present. This comparison will serve as a check on the accuracy of the apparatus.

The values given by the present equation for the average heat transfer from a vertical cylinder [equation (11) with $K = 0{\cdot}6$ and $m = 1/4$] are only some 2% higher than those given by McAdams [4] equation:

$$Nu_L = 0{\cdot}59(Gr_L Pr)^{1/4}. \qquad (14)$$

Equation (11) for the horizontal cylinder (with $K = 0{\cdot}158$ and $m = 1/3$) is based on L as characteristic dimension. With D as characteristic dimension it becomes:

$$Nu_D = 0{\cdot}508(Gr_D Pr)^{1/4}. \qquad (15)$$

The values given by equation (15) are only some 3·4% less than those given by McAdams [4] equation for horizontal cylinders:

$$Nu_D = 0{\cdot}525(Gr_D Pr)^{1/4}. \qquad (16)$$

For the inclined cylinder no quantitative comparison can be made with the available investigations as a result of the different ranges (or conditions) in which they were made. It is of interest, however, that all the previous investigations agree with the present results in trend.

CONCLUSIONS

As a result of the present experimental investigation the following conclusions can be made:

1 For the same flux both the local and the average heat-transfer coefficients increase with the angle of inclination of the cylinder. The minimum value occurs in the vertical position and the maximum value in the horizontal position.
2 The end of the laminar region, expressed as $Gr_x Pr$, increases with the angle of inclination.
3 The local heat-transfer results for all the angles of inclination can be represented by equation (10) and the average heat-transfer results by equation (13).

REFERENCES

1. E. A. Farber and H. O. Rennat, Variation of heat transfer coefficient with length—inclined tubes in still air, *Ind. Engng Chem.* **49**(3), 437–440 (1957).
2 M. Khamis, Studies in heat transfer by free convection from the outer surface of vertical and inclined cylinders to air, M.Sc. Thesis in Mechanical Engineering, Al-Azhar University, Cairo (1974).
3 P. H. Oosthuizen, Experimental study of free convective heat transfer from inclined cylinders, *J. Heat Transfer* **98C**, 672–674 (1976).
4 W. H. McAdams, *Heat Transmission*, 3rd edn. McGraw-Hill, New York (1954).

CONVECTION THERMIQUE NATURELLE ET LAMINAIRE AUTOUR D'UN CYLINDRE INCLINE

Résumé—On étudie expérimentalement la convection thermique naturelle et laminaire sur la surface extérieure d'un cylindre et chauffée uniformément (condition de flux uniforme), pour différents angles d'inclinaison du cylindre. Les équations générales sur l'effet de l'inclinaison sont déterminées pour le transfert thermique local et moyen.

DER WÄRMEÜBERGANG AN EINEM GENEIGTEN ZYLINDER BEI LAMINARER FREIER KONVEKTION

Zusammenfassung—Der Wärmeübergang bei laminarer freier Konvektion an der äußeren Oberfläche eines gleichförmig geheizten Zylinders (konstanter Wärmestrom) wurde experimentell bei verschiedenen Neigungswinkeln des Zylinders untersucht. Allgemeine Gleichungen für den Einfluß der Neigung wurden sowohl für den örtlichen als auch den mittleren Wärmeübergang aufgestellt.

ТЕПЛООБМЕН НАКЛОННОГО ЦИЛИНДРА ПРИ ЛАМИНАРНОЙ ЕСТЕСТВЕННОЙ КОНВЕКЦИИ

Аннотация — Теплоотдача в условиях ламинарной естественной конвекции от внешней поверхности однородно нагретого цилиндра (условие постоянства теплового потока) исследовалась экспериментально при различных углах наклона цилиндра. Получены общие уравнения как для локального, так и среднего теплообмена с учётом влияния угла наклона.